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Fluid Model of Plasma Outside a Hollow Cathode Neutralizer

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The present study analyzes the capability of a fluid model of electron transport to explain observed properties of the external plasma of a hollow cathode neutralizer used to neutralize a beam emerging from an ion thruster. Calculations reported here show that when the effective collision frequency in such a model is near the plasma frequency, the resulting electric potential and electron temperature variations are in qualitative agreement with values measured in the plume mode of the hollow cathode. Both theory and experiment show strong variations of temperature and potential within a few centimeters of the cathode orifice.

Nomenclature

E	= kinetic energy of an electron
$\mathbf{E}$	= electric field
$\mathbf{j}$	= net current density
k	= Boltzmann's constant
L	= scale length for variation of macroscopic plasma properties
m	= electron mass
n	= electron density
n_i	= density of ions (+) and electrons (−); $i = +, -$
p	= scalar electron pressure
q	= magnitude of electron charge
$\mathbf{q}$	= heat flux
$\mathbf{r}$	= position vector
$\mathbf{R}$	= collisional drag force between electrons and ions
T	= electron temperature
$\mathbf{u}$	$= \mathbf{V}_- \mathbf{V}_+$
$\mathbf{V}_-$	= drift velocity of electrons
$\mathbf{V}_+$	= drift velocity of ions
$\tilde{v}$	= jitter velocity of electrons
δn	= electron density fluctuation
η	= plasma resistivity
θ	$= kT$
κ	= thermal conductivity of plasma
κ'	= coefficient defined by power law dependence of κ on θ, $\kappa = m\,\kappa'\,\theta^{m-1}$, with m a pure number
λ_c	= mean free path for pair collisions between electrons
λ_D	= Debye length
ν	= effective collision frequency
ν_{ci}	= electron-ion collision frequency
σ	$= \eta^{-1}$
ϕ	= electric potential
ω	= frequency of oscillation of fluctuating field
ω_p	= electron plasma frequency

Introduction

THE purpose of the present study is to show that when an effective collision frequency near the electron plasma frequency is used in a classical fluid model of electron transport, the resulting electric potential variations and electron temperatures are in qualitative agreement with values measured beyond the orifice of a mercury hollow cathode neutralizer. The same conclusion had been reached in previous applications of the fluid model to the study of neutralized ion beams.[1]

It is proposed to explain the plasma properties observed in the aforementioned experiments in terms of anomalous resistance of the plasma to the flow of electron current. The calculations are based on fluid equations expressing conservation of charge, momentum, and energy. The classical (ignoring thermoelectric effects) form of the equations of electron transport is adopted,[2] but reduced values of the transport coefficients are permitted.

While the plasma is not a collision dominated fluid, randomization of electron velocities may still occur through enhanced levels of fluctuating fields, such as those initiated by streaming instabilities. The fluctuating fields are probably effective in coupling neutralizer electrons into the bulk plasma and in equalizing the mean drift of electrons with ions in the system. Such effects are often approximated by introducing an effective collision frequency, ν.

Competition between collective and collisional effects is controlled by the density. Within a few centimeters of the orifice the electron densities satisfy nL 10^{13} cm^{-3}, and their velocity distribution is characterized by temperatures θ between about 1 and 5 eV. The Debye length

$$\lambda_D = 743\sqrt{\theta/n} \quad \text{cm}$$

is typically small compared to distances $L \sim 1$ cm, over which there is a substantial variation of macroscopic plasma properties such as density, potential, and temperature. On the other hand, the mean free path for pair collisions

$$\lambda_c \approx 10^{12} E^{1/2} \theta^{1/2} / n^{-1} \text{ cm}, \quad E \lesssim \theta$$

for electrons of energy E (eV) is typically long compared to L, so that, as previously asserted, the behavior of the plasma is controlled by collective rather than collisional effects. Since $\lambda_D \ll L$, the plasma is quasineutral, departures from neutrality amounting roughly to

$$\delta n / n \sim (\lambda_D / L)^2 \sim 10^{-4},$$

the space around the neutralizer is strongly shielded from surface potentials. This is in contrast to the situation that prevails in charging of spacecraft in geosynchronous orbit, where effects of space charge are entirely negligible and potentials are determined as solutions of Laplace's equation.

Although collisionless, neutralizer and thruster-generated plasmas exhibit macroscopic behavior similar in many respects to that of a collisional plasma. Such behavior is perhaps not totally unexpected in view of the fact that in both nonequilibrium and equilibrium plasmas electrons are scattered by fluctuating electric fields. A primary difference

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between the equilibrium and nonequilibrium cases is the magnitude of the fluctuating fields.

Several investigators have measured properties of thruster-generated plasmas.[3-9] In the experiments of Sellen and Bernstein[5,6] and Ogawa et al.[7,8] on cesium ion beams neutralized by electrons from a hot wire, measurements were made of the density, potential, and electron temperature in the beam plasma. The potential difference between the neutralizer wire and the plasma could be varied by changing the position of the wire, the large potential differences (electron injection voltages) occurring when the wire was completely withdrawn from the beam plasma. An important result of the Ogawa experiments was that over a wide range of conditions electron density n and plasma potential ϕ were well correlated by the barometric law

$$n(r) = \text{const} \exp [q\phi(r)/kT] \tag{1}$$

The approximate validity of the barometric law was further verified by Kaufman.[9]

Siegfried and Wilbur conducted probe measurements of the plasma temperature and potential outside a hollow cathode operating in both plume and spot modes.[10] In the plume mode, the measurements show strong variations of potential and temperature within a centimeter or so of the orifice. Such results cannot be explained on the basis of a barometric law. We anticipate, however, that the observed behavior should be explained in terms of the anomalous resistivity of the plasma to the flow of electron current. Thus the primary objective of the following sections of the report is to determine the capability of simple transport models to explain, at least qualitatively, the experimental results.

The next section summarizes the fluid equations for the electrons in terms of conservation of charge, momentum, and energy and discusses the method of solution. Application of the equations to the exterior plasma of a hollow cathode and the results are discussed in the following two sections. The final section summarizes this study and the conclusions drawn from it.

Approximations for Electron Gas

Consider that the plasma is in a steady state and that quasineutrality pertains throughout the bulk plasma (that is, away from electrodes and collecting surfaces). The electrons and ions each satisfy the particle continuity equation

$$\nabla \cdot n_i V_i = 0 \qquad (i = +, -) \tag{2}$$

with $n_+ = n_- = n$. The momentum equation simplifies considerably if the electron drift velocity V is small compared to the random velocity and if the velocity distribution is nearly isotropic. Then, in the absence of magnetic fields,

$$\nabla p + qnE = R \tag{3}$$

In a classical plasma dominated by collisions, R is composed of a part proportional to the relative motion $u = V_- - V_+$ between electrons and ions, leading to plasma resistivity, and to a thermal part proportional to the gradient of electron temperature, which is frequently neglected. In this approximation,

$$\nabla p + qnE = \eta nqj \tag{4}$$

where

$$p = nkT = n\theta \tag{5}$$

is the electron pressure, j is the net current density, and the plasma resistivity η is related to the electron-ion collision frequency ν_{ei} by

$$\eta^{-1} = (\omega_p^2/4\pi)(1/\nu_{ei}) \tag{6}$$

If the plasma is nonresistive and isothermal, Eq. (4) yields the barometric law, Eq. (1).

If the plasma is not collision dominated, randomization of electron velocities may still occur through the enhanced levels of fluctuating fields in the plasma, such as occur for electron two-stream instabilities, or electron-ion instabilities of the ion-acoustic or Bunemann type.[11,12] These mechanisms are probably effective in coupling neutralizer electrons into the bulk plasma and in equalizing electron and ion mean drift velocities. They are often approximated by introducing an effective collision frequency, ν, in place of ν_{ei}.

The determination of electron temperatures in the plasma requires consideration of the energy balance equation. Making the same approximations in the equation expressing conservation of energy that were made in the momentum equation, yields

$$\nabla \cdot q = \eta j^2 \tag{7}$$

The heat flux q contains new features. Classically, q contains two terms: one proportional to the relative drift velocity between electrons and ions, and the other proportional to the gradient of electron temperature.[2]

For the initial calculations, we ignore the drift contributions to the energy flux and the electron-ion heating, and assume that the heat flux is proportional to the temperature gradient. The energy balance equation thus assumes the simple form

$$\nabla \cdot \kappa \nabla \theta + \eta j^2 = 0 \tag{8}$$

The theoretical basis expressed by Eqs. (4) and (8) is dependent on the existence of strongly fluctuating electric fields to randomize the velocities of electrons in the vicinity of the neutralizer. If such fields exist and oscillate at a frequency ω, the effective collision frequency seen by electrons would be of order

$$\nu \sim \omega_p \langle E^2 \rangle / n\theta$$

where $\langle E^2 \rangle$ is the mean square amplitude of the fluctuating field. In a classical collisional plasma,

$$\langle E^2 \rangle \sim n\theta / n\lambda_D^3$$

and ν reduces to the ordinary classical collision frequency. In general for a field oscillating at frequency ω, the jitter velocity of electrons is

$$\tilde{v} = qE/m\omega$$

so that

$$\frac{3}{2} n\theta \sim \frac{n}{2} m \langle v^2 \rangle = \tfrac{1}{2} n \frac{q^2 \langle E^2 \rangle}{m\omega^2} = \frac{1}{8\pi} \frac{\omega_p^2}{\omega^2} \langle E^2 \rangle$$

Thus, for $\omega \sim \omega_p$, the mean energy density of fluctuating fields and the mean particle energy density are comparable and $\nu \sim \omega_p$ exceeds ν_{ei} by a factor $n\lambda_D^3 \sim 10^3$. In this circumstance, the required condition of strong turbulence would exist near the neutralizer.

Equations (2-8) have been incorporated into a two-dimensional R-Z computer code. For the present applications, the ion density is a specified function of position and ion velocities are set to zero. The net current in the plasma is given by

$$j = \sigma[-\nabla\phi + (q/n)\nabla p] \tag{9}$$

The code determines electrostatic potentials by solving $\nabla \cdot j = 0$. It is necessary to iterate between this equation and the temperature equation [Eq. (8)], since the pressure is a function of temperature. On the various boundary regions, either isothermal or insulating boundary conditions may be specified. Since, in practice, we take κ to have a power law dependence on θ, $\kappa = m\kappa'\theta^{m-1}$, the equation actually solved is

$$-\nabla \cdot \kappa' \nabla (\theta^m) = j^2/\sigma \tag{10}$$

For convenience, the transport coefficients σ and κ' are calculated by a single isolated subroutine. The conductivity σ may depend on both density and temperature, and κ' on density only. The present version assumes a relaxation rate proportional to the plasma frequency so that

$$\sigma = n^{1/2}(e^2/m)(1/8.98\alpha) \tag{11}$$

where the parameter α is taken to be 0.51. By the classical Weideman-Franz law,

$$\kappa = \frac{3}{2}\sigma(k/q)^2 T \tag{12}$$

If we measure temperature in eV, $k = q$, so that $\kappa' = 3/4\,\sigma$.

Application to Neutralizer Plasma

Siegfried and Wilbur[10] conducted experiments to determine the density, electric potential, and electron temperature in the exterior plasma produced by a hollow cathode neutralizer. The experimental results together with essential geometrical features, discharge current I_D, discharge voltage V_D, and ampere equivalent neutral mercury flow rate $\dot{m}$ are given in Figs. 1 and 2. The plume mode discharge occurred between the hollow cathode and an anode a few centimeters downstream from the 0.75-mm-diam orifice of the hollow cathode. Measurements were also made for spot mode operation, but the corresponding theoretical analysis has not been performed.

For theoretical analysis of the plume mode results, a version of the code was constructed with 0.002-m radial resolution and 0.004-m axial resolution and applied to the region downstream from the keeper. Ion density, falling according to the inverse square law from a point source 1 cm behind the keeper, was normalized to a density of 10^{18} m^{-3} on axis in the keeper plane. A current of 2 A, assumed to flow uniformly through a 1-cm radius keeper, was distributed in proportion to the ion density at the downstream boundary of the computational grid. The electron temperature was set to 1 eV at the keeper plane in approximate conformance with the measured value; a null heat flux condition, $\nabla\theta = 0$, was applied at all other boundaries to simulate the effect of an electron repelling sheath.

The theoretical results (Figs. 3 and 4) show, in agreement with experiment, a sharp rise in both temperature and

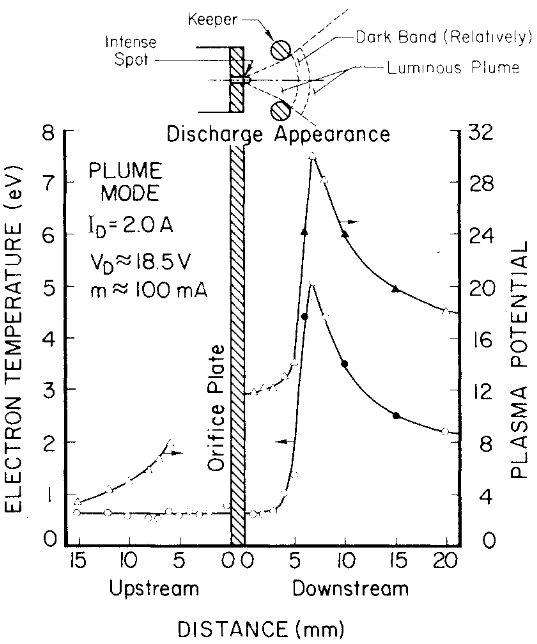

Fig. 1 Cathode plasma potential and electron temperature profiles—plume mode (from Ref. 10).

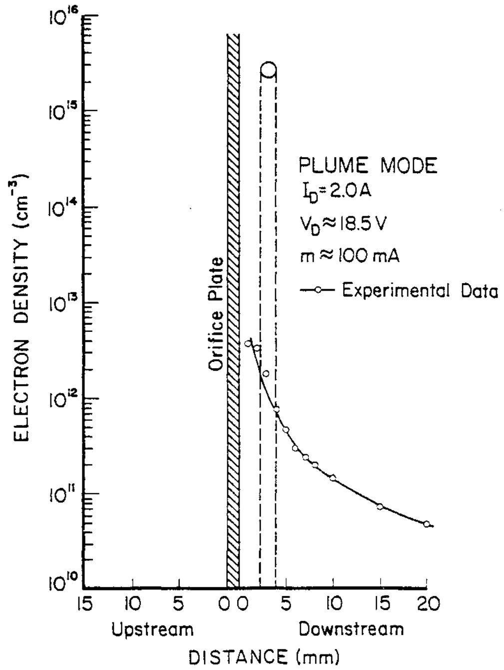

Fig. 2 Cathode electron density profiles—plume mode (from Ref. 10).

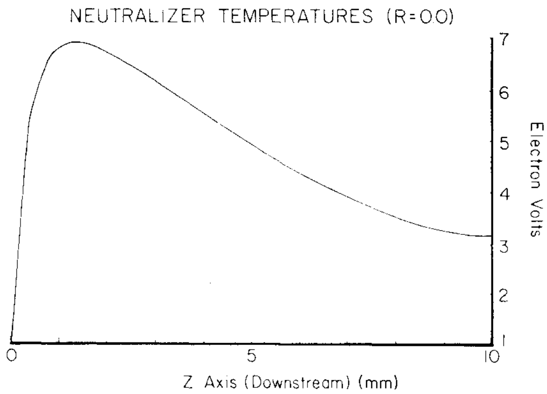

Fig. 3 Computed electron temperature on axis as a function of position downstream from keeper plane.

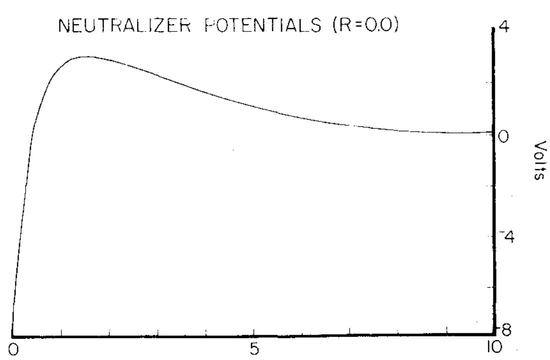

Fig. 4 Computed electron potential on axis as a function of position downstream from keeper plane.

potential, followed by a gradual drop as one proceeds downstream from the keeper. The quantitative agreement between theory and experiment is quite good. (Observe expanded scales along axes in Figs. 3 and 4.)

Discussion

Theory and experiment agree in exhibiting strong structure in temperature and potential profiles downstream from the orifice of the neutralizer. Theoretically, the spatial structure is obtained within the context of a quasineutral plasma; that is, the structure a few millimeters downstream from the orifice is not associated with Debye screening effects. Such screening effects should not be present, since owing to the high plasma density $N \sim 10^{12}$ cm^{-3}, the Debye screening length, $\lambda_D \sim 10^{-3}$ cm, is much shorter than the observed length scale of potential and temperature variations. Reducing ν by an order of magnitude below ω_p would lead to nearly flat temperature and potential profiles, thus destroying the agreement between theory and measurement. The spatial extent of strong turbulence within which $\nu \sim \omega_p$, is not known, nor are the conditions for its occurrence. Resolutions of such questions, requiring separate equations for the intensities of waves in the plasma, is beyond the scope of this study.

Effects of magnetic fields, which may be several Gauss near the neutralizer, have been ignored. This is justified in the present model since effective collision frequencies $\omega_p \sim 5 \times 10^{12}$ s are much greater than the electron cyclotron frequency.

It would be informative to carry out calculations without the specification of ion density as a prescribed input quantity. Such calculations would involve an independent determination of degree of ionization from appropriate rate equations and a knowledge of the flow rate of neutral mercury through the orifice. Whether or not such an augmented model would yield the observed plasma density and the transition from plume to spot mode at larger discharge currents has not been determined.

Conclusions

The consequences of the assumption that the electron gas in the exterior plasma of a hollow cathode neutralizer operating in the plume mode is effectively collisional have been analyzed. It is found, in agreement with previous studies,[1] that $\nu \sim \omega_p$ is required to give calculated plasma properties in agreement with measured ones.

In conclusion, further work should be performed to test the qualitative and quantitative capabilities and limitations of fluid models of thruster-generated plasmas, and to better understand the relationship between such models and the underlying plasma physical mechanisms embodied in the Vlasov equation.

Acknowledgments

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